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Using History to Teach Complex Numbers

Abstract

As usual we used the textbook approach to introduce the imaginary unit *i*, whose square equals to -1. When the class ended, a student asked if negative numbers could have logarithms. Does log(-1) exist? His question reminded us to reconsider how to make more sense of the imaginary unit *i*. There must be some more natural and intuitive alternative to define it. The historical development of mathematical concepts and the way that they develop in an individual mind are observed to be very much alike (Jankvist 2009). With an exploration of a history-based classroom practice and students' performance in non-routine problems solving, our research aims to examine when and how, and in which context to introduce the imaginary unit and what geometric intuition can serve to enhance students' understanding.

The case study took place with a class of 11^{th} grades students (16-17 year old) in our high school. The theoretical framework used to study the teachers' classroom practices is the double approach (Vandebrouck 2012). We articulate both the analysis of students' and teachers' activities in order to identify understand and interpret the link between the teaching of complex numbers using original sources and the corresponding students' activities. From a theoretical point of view, we attempt to answer the following two research questions:

1. Can students' own solutions to a cubic equation together with Cardano's attempt, Leibniz's doubt and Bombelli's discovery pave the way for the introduction of the imaginary unit *i*?

2. Can geometrizing the imaginary unit as done by Wessell, Argand, and Gauss, enable students to visualize the concept and acquire an intuitive understanding on dealing with non-routine, open-ended "real-life"" challenges thereafter?

The data that we collected consist of videos, classroom sessions, and interviews with students.

Our research indicates that teachers themselves first need to be well trained in the history of mathematics so as to better judge how students should acquire such knowledge. The concern about the logarithms of negatives raised by the student mentioned above can be encouraged to be an interesting after-class historical project. Meanwhile, we confirm that the utilization of original sources in the mathematics classroom can activate students' engagement to make mathematical discoveries, when different strategies flash into their minds and a repertoire of diverse representations are compared and the best is then chosen. Also, we address some obstacles to be overcome. In the presentation, a detailed teaching scenario with the historical package, teachers' implementations and students' activities as well as implications for teaching and research will be presented and discussed.

Questions discussed in our class include the following:

Question 1: Find the solution to the equation $x^3 = 15x + 4$.

Question 2: Calculate the above cubic equation with Cardano's formula; share what you discover.

Question 3: Leibniz considered the following situation: Let x and y be positive values, and $x^2 + y^2 = 4$ as well as $xy = 5\sqrt{5}$. Can you find the value of $x + y$ and x, y respectively? Explain why.

Question 4: With Bombelli's equation $\sqrt[3]{a+b\sqrt{-1}}+\sqrt[3]{a-b\sqrt{-1}}=c+d\sqrt{-1}+c-d\sqrt{-1}=2c$, can you give new insights to $x=\sqrt[3]{2+\sqrt{-121}}+\sqrt[3]{2-\sqrt{-121}}$ (this is what we got when we applied Cardano's formula to solve the cubic)?

Question 5: Compare with the number -1, what conclusion can you draw about the algebraic and geometric representation of the imaginary unit *i*?

Question 6: Share your ideas about Wessel's work with your classmates and infer the geometric representation of $\sqrt{-1}$ (Ever since Wessel, then, multiplying two directed line segments together has meant the two-step operation of multiplying the two lengths, with length always having a positive value, and adding the two direction angles. These two operations determine the length and direction angle of the product, and it is this definition of a product that gives us the explanation for what $\sqrt{-1}$ means geometrically).

Question 7: A treasure hunt with imaginary numbers (quoted from one, two, three,…, infinity by George Gamow.)

References

Jankvist,U.T.(2009). A categorization of the 'whys' and 'hows' of using history in mathematics education. *Educational Studies in Mathematics*,71(3),235-261.

Vandebrouck,F.(Ed.).(2012).*Mathematics classrooms:Students' activities and teachers' practices.* Rotterdam:Sense Publishers.

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