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On Mathematical Reasoning

Abstract

A common opinion is that we learn mathematical reasoning while learning mathematics. However, it is important to characterize explicitly what mathematical reasoning is. Hermann Weyl (1956, 1832) says that "mathematicians are no Ku Klux Klan with a secret ritual of thinking" which refers to a special kind of explicitness and transparency of mathematical reasoning. However, he continues by saying that we "should not expect me [Hermann Weyl] to describe the mathematical way of thinking much more clearly than one can describe, say, the democratic way of life", which entails that it is not easy to explicate the core idea of mathematical reasoning. The paper byWeyl (1956) is very important and profound. It characterizes several different aspects of mathematical reasoning and its applications.

To characterize mathematical reasoning we have to consider logic of mathematical reasoning from two different points of view. The first is how we prove mathematical results. Weyl (1956) focuses his attention to this. Let us call it just logic. In lecture courses of mathematics and mathematical logic this is the main topic, and hence we are acquainted with this sense of mathematical reasoning. However, the other point of view is the so called metatheoretical logic, or simply metalogic. Both points of view are of central importance, but in questions about the foundational character of mathematical reasoning the latter become more central. It is important to study what is the relationship between the two logics. Often, in the studies of mathematics education the focus has been, for example, in mathematical concepts and psychology (Ben-Hur 2006) or reasoning and communication (Berinderjeet& Toh 2012). In this presentation we focus on the attention to understand mathematical reasoning (i.e., metamathematics and its relation to mathematical (and logical) reasoning).

The need of metatheoretical logic becomes evident when we speak about the character of mathematical reasoning. Of course, practise of the mathematical reasoning is theorem proving and single computations, but all this does not characterize the foundational character of mathematical reasoning. The metalogic is, by definition, a key to understand the foundations of mathematics as demonstrated by the famous metalogical results, like the theorems of Löwenheim, Gödel, or Tarski. These metalogical results give information about mathematical reasoning and more generally about mathematics. Moreover, at the same time they give important information about the methodology of science. In fact, this allows us to see the connection between logic and metalogic (Hendricks 2007; Shapiro 2002). For example, from the metalogical point of view we can analyse the reasonability of structuralism in the philosophy of science. It seems that in structuralism the intention is to generate a metalogical framework without using logic explicitly. Structuralists identify logic with the first of the meanings above. So, by an explicit metalogical theory we can deepen our understanding of the methodology of science.

Metalogic is an important branch of mathematical study which has great theoretical importance in understanding mathematics and the methodology of science. This deepens our understanding about mathematics and mathematical reasoning (Usiskin 2015). Moreover, metalogical knowledge deepens our pedagogical understanding; it helps us to develop the teaching methods of mathematics but also of natural sciences (Sieg 2002; Koponen & Kokkonen 2014).

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