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## **Problems and methods in elementary geometry, according to Julius Petersen**

### **Abstract**

According to Julius Petersen in 1866, to do geometry is to examine a figure by propositions which are, either theorems (a certain well constructed figure satisfies certain conditions), or problems of construction (we require a figure so constructed, that certain given conditions are satisfied). So here “theorems” and “problems” do not mean exactly the same thing as in Euclid and Proclus: they are thought in duality.

By using straight edge and a pair of compasses, Petersen solved problems of construction of a point satisfying several conditions. Two ancient methods are usual for that: analysis by angles and lengths, with the tool of the three cases of equality of triangles taken as atomic properties to be combined; and analysis by equations on lines, with the tool of equational calculus. These methods are rather static. In contrast with these methods, thanks to the development of geometrical transformations especially during the 19th century, Petersen proposed a more dynamical method by determination of loci of points, and by transformations of figures. His method can be summarized in three rules: (1) To draw a figure representing the solved problem; (2) To imagine one of the given conditions for the figure in quest of removed, and seek the loci of the points of the figure thus rendered indeterminate; (3) Of the drawn figure try to form another, in which the relations between the given and sought elements are more convenient.

It is not completely new to draw a figure, to observe a locus of points as a curve, or to modify a figure: what is truly new here is the explicit determination of a method with this design of rules. And furthermore, Petersen proved that many of the problems previously solved by examinations of triangles or equations can be solve in this way. Finally, the very point of the method is that the book is not ordered as a theory, but it is ordered according to problems and as a practice of invention, according to difficulties of constructions.

According to works of É. Barbin, J. Lützen *et alii*, G. Moussard, we will consider Petersen’s book among other books for teaching geometry in the 19th century, showing how Petersen’s rules are in action in some problems.

This book is a splendid tool for teaching, noticeably for a self-taught-person, because it is truly a method: not only you know solutions, but also you are learned how to find them. So, it can be considered as a method to teach and learn geometry.

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