

**Uffe Thomas Jankvist**  
Aarhus University, Denmark  
utj@edu.au.dk

**Mario Aguilar**  
Instituto Politécnico Nacional, Mexico  
mario.sanchez@me.com

**Morten Misfeldt**  
Aalborg University, Denmark  
misfeldt@learning.aau.dk

### **Tschirnhaus' transformation: mathematical proof, history and CAS**

#### **Abstract**

The paper addresses the potential of using history of mathematics in combination with ICT to illustrate the notion of proof and proving to (undergraduate) students and relates such use to research findings regarding students' difficulties with proofs and proving (e.g. Dreyfus, 1999; Duval, 2007) as well as the role of counter-examples in relation to mathematical proof. In relation to the use of ICT, the paper draws on the instrumental genesis (Trouche, 2005), the distinction between epistemic and pragmatic mediations (Artigue, 2002), and Nabb's (2010) model for use of CAS in mathematics education.

Based on Alibert's (1988) construct of generating so-called scientific debates in the classroom, the paper outlines a teaching and learning activity, where CAS may play the role of a 'white box' and an epistemic mediation in relation to the notion of proof. The historical case used to illustrate this is Tschirnhaus' transformation from 1683, suggesting a presumed method for solving  $n$ -degree algebraic equations, drawing also on the correspondences between Leibniz and Tschirnhaus' concerning this transformation. From this point of view, CAS comes to support and illustrate Leibniz' arguments for the students.

**Keywords:** History of mathematics, original sources, CAS, mathematical proof.

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