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Philosophical and Didactic Practice in the Universe of fractions. Trace and Icon

Abstract

The long persistence of unsatisfactory results in teaching and learning fractions[1], the many subconstructs of the concept of fraction[2], and the inhibitions[3] caused by didactic processes limited to a single subconstruct, have led us to a philosophical and didactic practice concerning the concept of fraction. The reflective and disputative philosophical practice has started from the basic question: "What is the 'originary' content of the concept of fraction?"[4]. The didactic practice has investigated an initial concept of fraction, proposed in some primary school classes, that goes through the different subconstructs of the concept, aiming at a correct intuitive representation[5].

A reflection of historical type has led us to conclude that the 'originary' content of the concept of fraction is ascribable to the comparison of quantities[6]. Hence the starting point of our didactic practice is the following act of mathematisation: "The comparison between two quantities is a pair of numbers"[7].

This initial act produces a "split". It introduces two initial acts concerning the concept of number: counting is a number, comparing is a pair of numbers. This splitting breaks the categorical framing in which acts concerning the number teaching are usually blocked.

This initial act is "imposed". Despite its simplicity it is not a spontaneous consequence of the common observing and acting. The act of comparison is simple, as simple are the usual representations (objective, by squares and segments). Instead the non-spontaneous leap consists in a special symbolic representation: A;B = 9;5 ("the comparison between A and B is the pair of numbers 9;5). Just this representation keeps the trace of the didactic process that is being undertaken.

This initial act is "pregnant with significance". The initial act of mathematisation directs the subsequent didactic path. The idea of common unit kept in it, guides towards a rethinking of the usual language related to the concept of measure. The teacher is then forced to "suspend" (Bezos & Sierpinska, 2017) his knowledge on fractions and to carry on a process of interaction with children, in order to unveil the path that the initial act indicates. Along this path the various subconstructs emerge, up to attune in the Euclidean division. It is a path that possesses the properties of presence and absence: presence because the initial act provides inescapable indications; absence because the indications find reality in the specific actualization. This presence/absence has prompted us to make use of the name "trace", echoing the philosopher Lévinas.

The initial act leads to a linguistic mathematisation that involves the names "Whole", "Units" and "Quantity": their corresponding formulas are a safe reference for didactic activities but are not directly used; children do not know them but they grasp the corresponding "forms".

The mathematisation process is realized in the Euclidean division. This latter is the historical milestone that has created confidence to our approach. In our didactics, the Euclidean division is not lived by children as a formula to memorize; it is rather the "icon" of their learning: icon (a) as 'memorative' (mnemonic) synthesis of its own active history of learning; (b) as target towards which the actualization steps attune; (c) as opening meanings and then trace for future activity; (d) as medium between teacher and children.

Bringing the construction of the new didactic universe of fractions to an initial act reveals an attitude to

think of a principle pervading that universe. If the reaction to the "violence of the ontological approach" has shifted the attention to fragmentation[8] and to singularity[9], our effort tries to recover elements of universality. Universality is different from totality: a distinction that historically has appeared with the acceptance of plurality of universes; a plural character we referred to in the construction of the new didactic universe of fractions.

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[1] Already in 1978 Streefland pointed out how "The concept of fraction has manifested itself in education as a refractory one".
[2] The five Kieren's sub-constructs are: part-whole, quotients, measure, ratios, operators. Other possible subconstructs are: proportionality, point on the number line, decimal number and so on (Longoni, Riva & Rottoli, 2016).
[3] "Not only 'part of a whole' diagrams are possibly misleading, but, more seriously, … their use may well inhibit the development of other interpretations of a fraction …" (Kerslake, 1986). "… The idea of fraction-of-something stays in its primitive, intuitive state and functions as an obstacle to the construction of a systemically connected knowledge about fractions." (Bezos & Sierpinska, 2017)
[4] Our question arises as a development of the Davydov's question about the "object sources" of the concept of fraction. The work of Davydov about teaching quantity, multiplication and fractions strongly influences our proposal.
[5] Our aim is the familiarization (Davydov) of children of primary school with the concept of fraction. So we dwell not about theorization of this concept (Bezos & Sierpinska), but about its correct intuitive representation (Fishbein).
[6] Historical reflections led us to consider the Pythagorean comparison method; the anthyphairesis. This method of comparison did not last long. Its crisis came with the discovery of incommensurable quantities and its difficulties were contrasted by the effectiveness of the Euclidean algorithm. This latter overshadowed the anthyphairetic comparison, which was consequently forgotten (Fowler, 1979). In HPM 2016 we presented a schematic summary of this comparison procedure (Longoni, Riva & Rottoli, 2016).
[7] The subconstruct "ratio" is our starting point (we are referring to Kieren's classification). But while usually "Ratio is a complex concept, which demands a long-lasting learning process" (Streefland, 1984), historical reflection allowed us to make it elementary; elementary because turned to "the originary elements", but also by reason of the "lightness" with which children have lived the proposed acts: their answer has been quiet, serene, with adequate results.
[8] Modeling is the application of a fragment of mathematics to a fragment of reality (Israel, 2002).
[9] "The singular as knowledge actualized in activity" (Radford & Sabena, 2015).

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