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On Euler's formula -- between standard and non-standard analysis: An interpretation of Euler's *Introductio in analysin infinitorum*

Abstract

The workshop combines an interpretation of historical sources, specifically (Euler 1755, ch. 3) and (Euler 1748, §§132–134), with methodological investigations on the role of definition in mathematics. The basic truths of modern mathematics – the series expansions of e^x, sin x, cos x – are definitions of complex analysis, while in the 18th century they were proved as theorems. Strangely enough, for mathematical definitions are considered to be mere conventions, while theorems are to be qualified as true or false.

We know that it was Euler who proved the identity $e^x = \cos x + i \sin x$. In his 1748 *Introductio in analysin infinitorum*, chaper VIII, he expanded sin x and cos x into series. In the Workshop, we focus on §§132-134, where Euler expands the function cos x. In his development, Euler does not apply any technique of real analysis, not to mention the notion of limit, but rather infinitesimal and infinitely large numbers instead. That is why we provide an interpretation of Euler's proofs in modern nonstandard analysis. Of course, it is out of question that Euler did not know the ultrapower construction that we apply, or any other technique of modern mathematical logic. He also ignored the 19th century real numbers, as well as the 20th century notion of an ordered field. However, we can show that he implicitly applied rules of non-Archimedean fields, and his operations with infinite sum can be interpreted in terms of nonstandard analysis. As an easy introduction into a technique of algebraic interpretation of historical texts, during the Workshop, we will provide an interpretation of Euclid's *Elements*, book V definitions 4 and 5, and Descartes' operations on line segments as introduced via diagrams in his 1637 *La Géométrie*.

This material and the associated interpretation have been used in my teaching, and students found this approach to ordered fields very instructive, especially because of the idea to introduce real numbers as a special ordered field, rather than via a construction. Mathematical curricula are different in different European countries, but it seems that they share a common feature; namely, that the notion of ordered field appears implicitly, rather than explicitly. Now, starting with the notion of an ordered field, one can grasp easily the idea of a non-Archimedean field. However, this idea is commonly overlooked in most educational systems. Therefore, this workshop is focused on the idea of an ordered field and its history. Seen in historical perspective, mathematical definitions are not simply conventions, but rather basic truths that cannot be proved. Thus, sometimes they turn to axioms; vide Euclid's *Elements*, V, def.4-5, or Descartes' definition of multiplication and division of line segments. In this connection, Euler's formula is a very special case, because in his development these expansions are proved. However, when the framework, i.e. when the non-Archimedean field has been replaced by the conventional ε-δ one, Euler's findings are introduced as definitions.

The general plan of the Workshop, as well as an annotated PDF file containing excerpts from historical texts is included.

Remark (by the Organizers): Additional material has already been uploaded on the ESU-8 website.

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