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Leonhard Euler's Differentials: An Attempt to Restructure Teaching of the Derivative Concept

Abstract

This workshop will present a strategy for teaching the derivative concept based on Leonhard Euler's text *Foundations of Differential Calculus.* The idea of using Euler's approach in a modern classroom is an extension of the primary source project (PSP) *The Derivatives of the Sines and Cosine Functions* (Klyve, 2017), itself developed as part of a larger effort to incorporate primary sources into the teaching of standard mathematical topics currently supported by major funding from the US National Science Foundation. In this particular PSP, Euler's method of determining differentials for the sine and cosine functions is shared with students through guided reading of selected excerpts from an English translation of Euler's original text. However, Euler's use of differentials is not limited to these two functions but is developed and applied to other functions throughout his text. In other words, the symbols *dy, dx* and the mathematical concepts these symbols represented had a central role in Euler's calculations.

In contrast to Euler's approach, the topic "differentials" is placed at the end of the chapter on derivatives in today's calculus textbooks. When the symbols *dy* and *dx* appear in calculations prior to this point in the course, they are used to represent the derivative function: $f'(x)=dy/dx$. This raises a number of perplexing questions for students and instructors alike:

Is [dy/dx] *a fraction, or a single indivisible symbol? What is the relationship between the dx in dy/dx and the dx in* $\int f(x)\,dx$*? Can the du be cancelled in the equation* $dy/dx = (dy/du)\cdot(du/dx)$*?* (Tall, 1993, p. 5)

The conceptual challenges encountered by students as a result of the current approach to teaching and learning calculus suggest that placing differentials at the center of the subject – as Euler did – might inform (and reform) our pedagogical approach to teaching not just about the derivative, but about calculus more generally. Such a pedagogical approach is currently being tested in a university-level first-year calculus course in the United States.

During the workshop, participants will consider several of its student tasks, the primary goal of which is to use Euler's perspective on differentials as the building block for student learning of the derivative concept. Following a brief introduction to a study conducted in Fall 2017 study, participants will first be asked to work together in small groups on these problems as calculus students might (approximately 15 minutes). They will then be asked to reflect on their problem-solving experience as "students" from the perspective of

a teacher, and discuss pedagogical issues and concerns in these same small groups (approximately 20 minutes). This will be followed by a whole group discussion during which participants will share their reflections and consider issues related to classroom implementation of the approach (approximately 30 minutes). Finally, we will share some findings from the actual classroom implementation of the PSP and other Euler-based differential tasks, and invite questions and reflections from the workshop participants (approximately 25 minutes).

References

Euler, L. (1755). *Foundations of Differential Calculus*. (J. D. Blanton, Trans.). New York, NY: Springer-Verlag New York, Inc. (Original work published in 1755)

Klyve, D. (2017). *The derivatives of the sine and cosine functions*. Retrieved from http://digitalcommons.ursinus.edu/triumphs_calculus

Tall, D. (1993). Students' difficulties in calculus. In C. Cauglin et al. (Eds.), *Proceedings of Working Group on Students' Difficulties in Calculus, ICME-7* (pp. 13–28). Québec, Canada.

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