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An Advanced proof of Pythagorean Theorem using artifacts and dynamic geometry software inspired by Montessori Method

Abstract

Recent advance in cognitive neurosciences is confirming that learning processes of mathematics and geometry happen in the same regions and circuits of the brain where people process information about space, time, approximate amount and number awareness. While learning mathematics, language is ordinarily used, but mathematical reasoning itself seems to happen in specific parts of the brain. These considerations are fundamental to conceive educational strategies to make mathematics and geometry easier to learn for pupils.

In our work we show the results of a teaching experiment in a primary school, realized using the Montessori method and setting mathematics education through perceptual-sensory inputs, especially by touch sense and hands and visual representations. Montessori's approach is founded on original principles: teachers must start from "things", that is, from concrete representations of geometrical objects. But, above all, with a self-discipline that is not easy to learn, teachers have to let the things themselves speak to the students.

Among the main features of the Montessori method, there are respect for individuality of each child, hands-on experiences, skills in cooperative and manual work. Therefore, this method is one of the possible instances of Pedagogical Activism: each pupil, if properly taught, may become an active constructor of his own knowledge.

Real and virtual tools were exploited to introduce pupils to one of the most important deductive process: the geometric proof.

Indeed, to achieve this goal, different artifacts were realized using everyday and cheap material, e.g. coloured frames, tiles, golden pearls, and many other classical tools, as "regoli", in order to explain and demonstrate the Pythagorean Theorem. For an instance, a first real proof was designed using a hydro-mechanical system, relying on a fluid contained in three squared boxes, two built on the two perpendicular sides of a right-angled triangle and the third built on the hypotenuse (see Fig.1).

Then a second real proof, based on cut out of shapes on a metal plate, was presented to pupils (see Fig.2).

In this way, artifacts give a visible proof of the equivalence of the areas.



Fig.1: Hydro-mechanical proof of Pythagorean Theorem

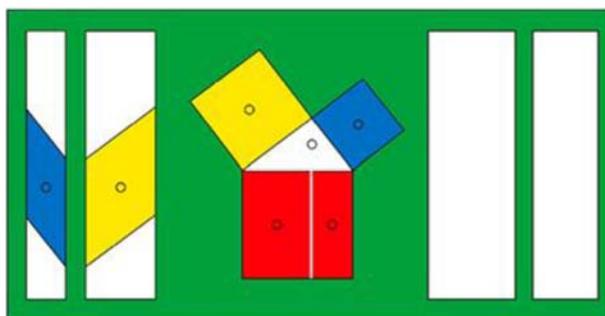


Fig. 2: Cutout proof of Pythagorean Theorem

Therefore a back track path was treaded: the theorem was not demonstrated as in traditional proofs, but starting from the opposite point of view - practice, direct observation by real and virtual tools, then only at the end, a thorough formulation of the theorem.

Besides, latest interactive and dynamic geometry software are nowadays introducing new practices in mathematics education, supporting traditional artifacts and compass-and-straightedge constructions with virtual but very effective 2D or 3D graphs, geometric shapes and animations.

So, a specific user-friendly and intuitive dynamic geometry freeware, GeoGebra, was used, eliciting enthusiasm and curiosity in the pupils: they built the shapes, determined the areas and so it was very straightforward to verify the equivalence between the squares. Furthermore, they repeated the proof of the theorem using real graphics tools, producing a picture full of creativity. Finally, we present a short comparison between technological innovative virtual tools and traditional real tools, such as ruler and compass. Teacher's task was to motivate the students, involving them in an education project that refers to an effective integration of both real and virtual tools.
