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Time Measurement as an interdisciplinary subject in Mathematics Education: The Calendar

Abstract

Unlike space, time can be conceived and delimited only if represented in terms of symbols, which themselves require and/or are susceptible to different interpretations. As a result, understanding time (more than understanding space) implies the imperative need to rationalize its representation in terms of arithmetical or geometrical symbols of a most unequivocal meaning. Perhaps this is the generic element underlying the interrelation between the concept of time and its mathematical elaboration. Therefore, as a fundamental formative category of human thought and perception of the world, conceiving time is indissolubly connected to its quantification through measurement, which in turn is realized by focusing on **periodic** (cyclic) phenomena.

On the other hand, inherent to the concept of measurement is the act of comparison: to compare objects with respect to a certain characteristic they possess in common, by agreeing to choose and choosing one among them as a standard of comparison; the unit of measurement of this common characteristic. This is true for any kind of measurement (related to physical, biological, economic etc. phenomena). In the case of time, it presupposes the existence of periodic phenomena **compatible** with each other; or in a more formal language, periodic phenomena for which the ratios of their periods do not change in the course of human life, or society's existence. And it is a fundamental **empirical fact** that phenomena do exist for which this condition is (approximately) valid.

From a historical point of view two points should be particularly stressed in this connection:

- It was first perceived and understood that periodic astronomical and physical phenomena compatible in the sense explained above do exist. Therefore, they were preferably selected for **quantifying** (hence, measuring) time.
- It was gradually (and slowly) realized that this compatibility was only **approximate**; actually that the more human observations and experimentations were becoming finer, the more rough this approximate compatibility was getting[1]. This development was strongly interrelated with the development of the required mathematics.

Because time (as a quantified concept) is so deeply rooted into our (modern) civilization, its accurate measurement[2] seems to be a subject elementary from a conceptual point of view. Therefore, its emergence and the way it has influenced deeply many intellectual, practical, political and religious aspects of human history is hardly appreciated in general, and in the context of education in particular. In fact, measuring time is closely related, touches upon and addresses interconnected questions and problems in astronomy, physics, technology, navigation, theology, politics and philosophy, most (if not all) of which were related to mathematical issues, each time raising questions and problems that finally led to important developments both in mathematics and the above mentioned disciplines (see e.g. Whitrow 1988, Fraser 1987, Borst 1993, Duncan 1998, Richards 1998).

Seen from an educational perspective, exploring this subject and in particular the various forms the calendar took in the course of history and in different cultures, constitutes a multifaceted, strongly interdisciplinary teaching module touching upon a variety of subjects. Or, aspects of this subject provide

insightful examples that in an elaborated form could illuminate and reveal the decisive influential role of (nowadays considered classical, elementary, or even trivial) mathematics[3] in addressing and tackling problems in the above mentioned disciplines and shaping man's ever-changing view of the world.

In this presentation

- An outline will be given of some of the main issues related to the problem of time measurement and the establishment of a widely accepted calendar in the course of history, as well as its significance in the context of a variety of disciplines, with emphasis on the underlying mathematical issues (ranging from the development of positional number systems, to the use of coordinates in cartography, and the development of spherical trigonometry).
- From a didactical point of view, this subject will be placed within the theoretical framework concerning the role of history and epistemology in mathematics education, presented in recent publications (Tzanakis 2016, section 3; Clark *et al.* to appear).
- Indicative examples will be given of the strongly interconnected, deeply interdisciplinary issues related to time measurement[4] and its didactical implications, with emphasis on the associated mathematics.

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[1] E.g. that the annual revolution of the sun was originally taken to be 12 lunar cycles; or later on, that it was 365 days etc. Due to the complexity of these periodic phenomena it took quite a long time to realize that this was not a good approximation for long time intervals, and to find a better approximation of their relative periods (see e.g. Auerbach 1995).

[2] In the sense of comparing as accurately as possible the periods of different cyclic phenomena, thus specifying "stable" units of time.

[3] E.g. the use of positional number systems; also, elementary concepts and results about congruences in number theory, continuous fractions etc. (see e.g. Rickey, 1985; Eisenbrand, 2012; Beveridge, n.d.; Grabovsky, n.d.).

[4] E.g. it is worth noting that the word "computation" comes from the Latin "Computus" signifying (since the 6th century AD) the calculation method to determine the calendar date of the Christian Easter (*Computus Paschalis*; see Borst 1993, ch.4); a theological problem that acted as a catalyst in the development of our modern concept of time, its measurement, and effective methods for doing complicated numerical calculations.

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