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**The concept of space in the history of mathematics and in the history of painting**

**Abstract**

In this presentation I will discuss the notion of space as it is used in paintings. I will start with the Renaissance painters Lorenzetti, Masaccio, Ucello, Leonardo, and Duerer, continue with Mannerists Tintoretto, Parmigianino and El Greco, followed by Baroque painters Velazquez, Rembrandt, and Pozzo, then Turner, the Impressionists like Monet and Renoir, post-Impressionist Seurat and Cezanne and will end with Cubists Picasso and Braque and Abstract painters Malevich and Kandinsky.

In analyzing the paintings I will concentrate on the construction of the spatial illusion, the point of view from which the painting is constructed, the way in which a contact with the spectator is introduced and the particular geometric tricks (as the splitting of the horizon in Mona Lisa, the use of a mirror by Velazquez, or deformations by El Greco) used by the painters. It seems that very similar changes like those found in painting, occurred on a more abstract level, in geometry as well. Thus, showing and discussing the particular changes in the history of painting, where they are clearly visually detectable, may contribute to the understanding of these changes in the development of geometry, where the abstract setting sometimes hides the nature of the epistemic shift.

Thus I will discuss the connection between the space used in the Renaissance paintings and projective geometry. In geometry the projection happens between a 3D object and its 2D image and thus is comprehensible in a realistic manner. In projective geometry the 3D object is replaced by its 2D picture and thus the projection is between two images, which is more difficult to comprehend. Similarly I will discuss the relation between Impressionism and the Erlanger Program of Felix Klein. In Impressionism the distinction between the neutral visual basis and the structure that we introduce into it can be directly perceived. In Klein the neutral basis is the projective plane and the structure introduced into it is the metric structure of the particular geometrical system. But in a deep sense they do the same thing - detach the neutral visual basis from the structure. It seems that this detachment can be more easily understood in the context of the Impressionist paintings than in the abstract setting of the Erlanger Program. A similar relation exists between Cubism and combinatorial topology. Just like Cubism, combinatorial topology aims at learning to represent objects independently of the surrounding space. Cubists do it by cutting the objects into cubes and rearranging these. Combinatorial topology uses instead decomposition into simplexes, but the basic aim and idea are similar. Once again the basic idea of liberating geometric objects from their dependence upon visual space is common to painting and geometry, while painting is technically more accessible.

I use this material in my courses on the history of geometry to illustrate the sometimes rather abstract geometrical theories as non-Euclidean geometry or combinatorial topology. It is important to realize that the very notion of space did not originate in the geometry of Euclid, but was introduced into geometry from treatises on painting, written by artists like Alberti or Dürer. I have the experience that painting makes possible the discussion of the fundamental ideas of geometry in a non-technical way, which improves comprehension and helps also to remember these deep ideas. Thus one may say that the end goal is a conceptual understanding of geometrical abstraction and building connections between different geometrical

theories as projective geometry, algebraic topology and non-Euclidean geometry.

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