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### **Research on factors affecting mathematics teachers' HPM Lesson Study**

#### **Abstract**

A Lesson Study (LS) is a practice-based, research-oriented, collaborative mode of professional development (Huang & Shimizu 2016), so we usually use Chinese LS to conduct HPM lessons at elementary and secondary schools. However, even though teachers conduct LS with support from professional learning community (PLC), they usually encounter some difficulties from other aspects. As the problems teachers meet in teaching constitute an important aspect for pursuing an investigation on the effectiveness of history in the classroom (Barbin 2000), it is necessary to investigate the issue deeply. Tzanakis & Arcavi (2000), Clark, et al (2016) sum up arguments against the incorporation of history, including at least two sources of difficulty: philosophical and practical. Clark and Lisa (2013) think the greatest challenge in planning lessons informed by the history of mathematics is to deal with the available resources. Poh & Dindyal (2016) describe similar difficulties to those mentioned by Clark & Lisa in developing a lesson package infused with the historical development of calculus by using an LS approach. But they do not mention factors stemming from different members of PLC, especially in China, where the community consists of a school-based group, a teaching experts group, an HPM academic group. So the research question in this communication is: what are the factors that influence teachers' HPM LS?

We use Wang, Qi and Wang's (2017) procedure of HPM LS, constructed from Gu and Wong's Keli Model (one popular type of LS in China ) (2002), and built on theories of HPM in China to conduct teaching of "Double digit divided by multiple digit" and "Parallel" in grade 4 in Shanghai. Four procedures are included in this model: Selecting a Topic & Preparing, Discussing & Designing, Implementing & Evaluating, Analyzing & Writing, and there are cycles among the former three procedures based on feedback of an implementation of teaching. The implementation of teaching before demonstrating is called rehearsal teaching, and rehearsal teaching is repeated multiple times until the teachers involved feel satisfied with the goal they set out to achieve (Huang & Bao, 2006). We collect lessons plans, videotaped lessons, post-lesson debriefing, the discussions among members in PLC, the teacher's and HPM experts' design feedback worksheet and teaching feedback/reflection worksheets, the pre- and post- questionnaires about students' learning, interviews after two lessons. We use a 3-level coding to organize data and use them as evidence to descriptive research. Based on the data after coding and interview, we form a report with 6 parts. The following is a sketchy description of each part based on the case of one teacher.

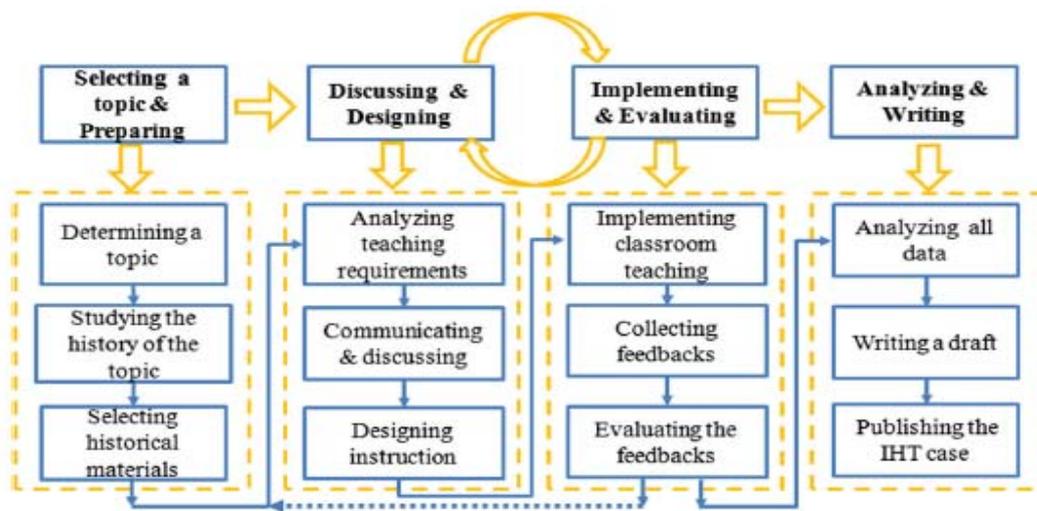


Fig.1 Wang, Qi and Wang’s procedure of HPM Lesson Study

**Part 1.** Teacher M firstly designed teaching herself. The historical resources she used are “Shang Shi Fa” in Ancient China (Fig.2 5984÷16) and evolution of long division in Western society since 17<sup>th</sup> century (Fig.3 732÷6), and M regarded them as exercises once students know how to calculate division. The purpose of the former is to make students understand culture in ancient China; the latter is to improve students’ ability to understand. After being provided with other resources and advice, M slightly revised her design, adding the idiom “Ban Jin Ba Liang ” (be six of one and half a dozen of the other) in background of weight units in the book “Yu Zhi Shu Li Jing Yun” (Empire Kangxi, 1722), and attended a plan meeting for communicating & discussing. In the meeting, she accepted advice (using two lines in design: one is relationship comprehending, building the relationship between “quotient is one-digit” and “quotient is two-digit”; one is background, using “Ban Jin Ba Liang” under a theory of variation”) put forward by the HPM academic group with pleasure.

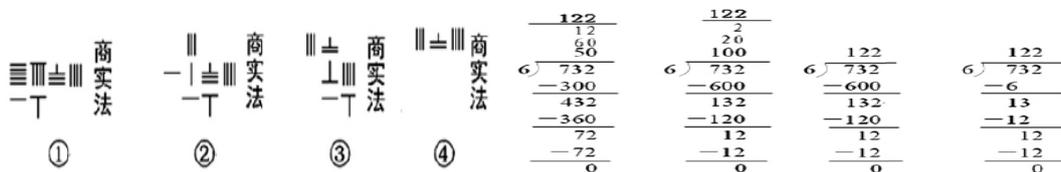


Fig.2 “Shang Shi Fa” in ancient China      Fig. 3 The evolution of division (Eugene,1982)

**Part 2.** After communicating with a teaching expert in her school, she revised the design to cater to the HPM academic group and the teaching expert. At the beginning of the design, M used “Zhi Chu Fa” (a method used in ancient China; namely to use subtraction in order to calculate division), and did not use the background “Ban Jin Ba Liang” in the whole process, as the teaching expert mentioned “The weight units used today (1Jin = 10Liang, 2Jin = 1kg) are different from these (1Jin = 16Liang, so half 1 Jin = 8Liang) in ancient, students would mix them up. I was surprised to read the design, which was different from what we had discussed, so I invited M and one expert in the HPM academic group to fill in a design feedback worksheet, and send the expert’s worksheet to M to make her change design, but I failed”.

**Part3.** M conducted rehearsal teaching using her design, but the outcome was not good, and the teaching expert and her colleagues’ comments in post-lesson debriefing made her sad. But M did not adopt the HPM academic group’s advice. Once more she slightly revised her design and taught again, but the outcome was still not good.

**Part 4.** M lastly decided to adopt the HPM academic group’s advice. First, she used idioms to build links between Chinese and Mathematics, which are thought to be nonexistent or only few. Second, she introduced an exploratory task, which is adapted from a problem in “Yu Zhi Shu Li Jing Yun”, to explore the relationship

between Jin and Liang in ancient China. Then assigning the task to find the links between Zhu and Liang, which is also adapted from a problem in “Yu Zhi Shu Li Jing Yun”, to introduce the idiom “Zi Zhu Bi Jiao” (haggle over every ounce 1liang=4zi, 1zi=6zhu). In exercises, M used True or False Questions (the third calculation method in Fig. 2 is included) to introduce the evolution of division (Fig.2), played micro-video to introduce “Shang Shi Fa”, and put all methods together to compare their advantages and disadvantages.

Part 5. Through “Double digit divided by multiple digit”, M gained some experience and tried to transfer these experience into “parallel”. She firstly attended a plan meeting, introducing her analysis of the textbook, historical sources and her students’ cognition. Through communicating & discussing with the HPM academic group, she decided to integrate the definition of Parallel in Euclid’s (300BC) “Elements” (Definition 23: Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction) and in Mohism “Mo Jing” (388BC) (“Ping, Deng Gao Ye”, meaning that if the distance between two lines is equal, the lines are parallel), idiom “No straightedge and compasses, no square and Circle ” (Means “Nothing can be accomplished without norms or standards”) into her design. Secondly, M finished her design. We were surprised of her good work after reading the design, although we put forward some advice. Her design included 6 links: (1) Introducing from learned knowledge “Intersection or no-intersection of two lines”; (2) using intuition to introduce the definition of parallels in Euclid’s “Elements”, using visual illusion of graph to trigger students’ consideration into the limitations of Euclid’ definition, and introducing definition in textbook “If two lines are perpendicular to the same line, they are parallel”; (3) using many tools to draw two parallel lines and students’ method to introduce Mohism’ definition, playing micro-video to introduce drawing tools in ancient China and the idiom “No straightedge and compasses, no square and Circle ”; (4) Before introducing notations of parallel in history (Fig. 4), students take part in activity of creating notation of parallel, and through comparing the notations in history and notations they create, students know characters of notations, for instance uniqueness and universality; (5) experience usefulness of parallel in real life. (6) Summary.



Fig. 4 The notations of parallel in History (Cajori,1993)

Part 6, through 4 trial teachings and post-lesson debriefings, discussions with members in PLC, revision and re-revision, M demonstrated her lesson successfully.

Combining the report of descriptive research with interview after the lessons, we find the teacher’s self-efficacy and teaching feedback are the decisive factors influencing the HPM practice. Besides, teaching experience, teaching experts, teaching researchers, colleagues and the HPM research group also have different impacts on the individual teacher’s HPM practice.

## References

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