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Discovering Neglected Synthetic Geometry on "Social Networks": Learning Maths as in the Historical Italian Academies

Abstract

There were about 600 Academies in Italy in 1525-1700 years: they were fundamental to the development of the intellectual networks and to the dissemination of ideas in Europe. In these academies, multidisciplinary paths were developed, for instance from the humanities, to the figurative and performance arts, natural sciences, medicine and mathematics.

In mathematics, there was a fervent discussion about "new geometry", i.e. *calculus* vs "old" synthetic geometry. Because of calculus' success, some very interesting results about synthetic geometry were neglected.

The aim of the present work is to learn about some hidden historical theorems of synthetic geometry developed during those years, such as theorems about "Cevian, Orthic and Pedal Triangles". These theorems are quite interesting from a mathematical and physical point of view, too.

In particular, Ceva's theorem is a necessary and sufficient condition for three cevian segments in a triangle to meet at a point. Heights, medians and bisectors are cevian segments, so verification of Ceva's hypothesis is sufficient to demonstrate the existence of barycentre, orthocentre, incentre, instead of different theorems.

The main interest in these theorems is around Fagnano's problem, demonstrated by using both calculus and geometry: the orthic triangle (which is the pedal and the cevian triangle of the orthocenter of a given triangle), with vertices at the base points of the heights of a given triangle, has the smallest perimeter of all triangles inscribed into an acute triangle.

Both Ceva and Fagnano theorems have been generalized, under some conditions, to orthic quadrilaterals of a convex quadrilateral. Fagnano's problem has been also applied to "polygonal billiard" physics. Great mathematical and physical interest in these topics is also due to Yakov G. Sinai, who was involved in ergodic theory in billiards and won the Abel prize in 2014.

In order to re-discover these hidden theorems, role playing, and drama methodology have been used. In particular, this innovative methodology has been experimented in three classrooms of Secondary Schools in the south of Italy. Italian Academies have been simulated and our students also "founded" a new modern Academy, named "Baffling Triangles", with a slogan ("Triangle: a universe letter") and a "logo", all

students wearing a black hooded sweatshirt. CLIL methodology has been also used in this simulation, because English is a universal language, instead of Latin used in the past. Nevertheless, some historical documents (e.g. Ceva theorem) have been studied in Latin, to critically analyse original fonts.

Information Technologies can provide significant advantages in this learning – teaching approach: they have been used to share and communicate main results, to create "virtual Academies" and to realize multimedial materials, too, which are "modern fonts" to be archived on cloud softwares. Nevertheless, "Social Networks" are commonly used by students, since they are "Digital Natives".

Last but not least, some collaborative web 2.0 resources, e.g. blogs and wikis, have been also used, in order to support collaborative learning, to share ideas and work groups, and finally to improve writing skills of pupils.
